Adjustment Methods for Measurement Error

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Adjustment Methods

- We should always aim to improve data collection processes to avoid measurement error
- When that is not possible, we can (and should) adjust its impact
  - This enhances the rigour of our research
  - And allows us to analyse data that would otherwise be too dubious

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- Ex.1, the effect of self-reported anxiety on life satisfaction (both of them subject to classical errors)
  - the reliability ratio can be derived by repeating the interview for a subsample of participants,
  - which can then be used to adjust the expected bias (assuming a simple linear model),

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- Ex.2, the effect of immigration on crime recorded by the police (systematic multiplicative errors)
  - the under-recording can be estimated using victimisation surveys,
  - and we can adjust the estimate of interest accordingly (assuming a linear model),

$$\widehat{\beta}^* = \widehat{\beta}/\bar{U}$$

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  - Instrumental variables (two stage processes)
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- Question: Could you use any of these methods for the measurement problems you have encountered in your research?
  - Validation and repeated observations are hard to find when you rely on secondary data

Adjustment Methods

- We will focus on methods that can be used without additional data
  - Simulations (*RCME* Pina-Sánchez et al., 2022)
  - SIMEX (Cook & Stefanski, 1994)
  - Bayesian adjustments (Gustaffson, 2003)
- All we need is an intuition of the form and prevalence of the measurement error

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  - Interviews with survey interviewers, experts (e.g. practitioners), or individuals from the target population
  - Our own educated guess as subject experts

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- Such estimates should be taken as highly uncertain
  - ‘Gold standard’ measures are rarely perfect
  - Problems of transportability with studies using different samples/populations
  - Subjective nature of qualitative methods
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- We should undertake multiple adjustments
  - Using a range of plausible values, as opposed to assuming we know the form and prevalence of measurement error mechanism/s perfectly
- We will not obtain a single ‘adjusted’ finding
  - Rather, we will seek to assess how ‘sensitive’ or robust our findings are under different scenarios
  - This is known as sensitivity analysis

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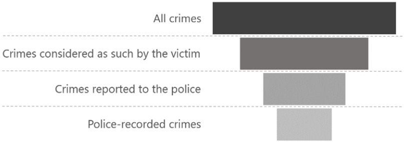
Simulations

Simulations

- The idea is to use our understanding of the measurement error process to recreate the original variable
- Then repeat the analysis using the 'adjusted' variable
  - Ideally for a range of measurement error scenarios
- Examples:
  - The reporting rate of burglaries has fluctuated between 40% to 60% in England and Wales (Pina-Sánchez et al., 2022)
  - Men report an average 14 lifetime opposite-sex partners, women report 7 (Mitchell et al., 2019)

Simulations: Under-recorded Crime



- We formalise the above intuition into a measurement model
  - $X^* = X \cdot U$ with $U \sim N(0.5, \sigma_U)$
- We rearrange the measurement model and substitute to adjust the error-prone variable
  - $\widehat{X} = X^*/0.5$

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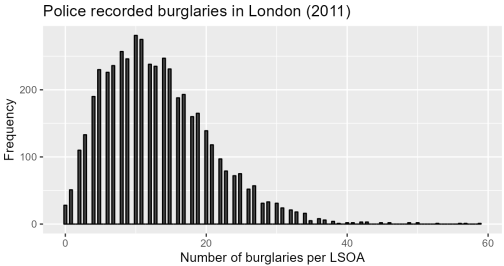

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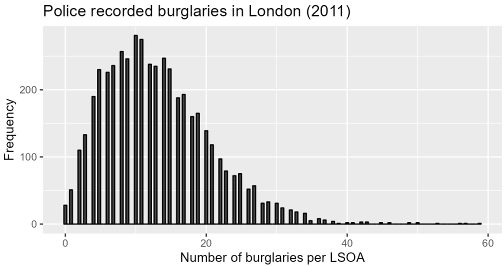

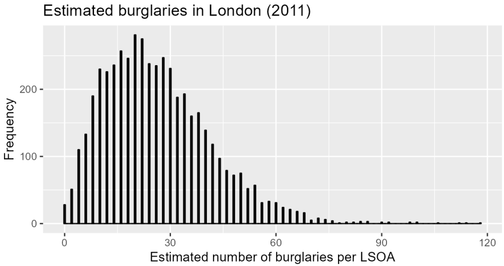

Simulations: Lifetime Partners

- A slightly more complex measurement error mechanism
- If we assume the true number of partners is in the middle (i.e. men overreport as much as women underreport)
  - We have the following measurement error model

$$\begin{cases} X^* = X \cdot U_1; & \text{if Z=man} \\ X^* = X \cdot U_2; & \text{if Z=woman} \end{cases}$$

  - And the adjusted variable

$$\begin{cases} \widehat{X} = X^*/1.33; & \text{if Z=man} \\ \widehat{X} = X^*/0.66; & \text{if Z=woman} \end{cases}$$

  - With the 33% worked out for men as: $14/(14-(7/2)) = 1.33$ and similarly for women: $7/(7+(7/2)) = 0.66$

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  - Many other adjustment methods can only be used in specific outcome models, or estimation methods
- They are also remarkably flexible in that they can mimic a wide range of forms of measurement error and misclassification
  - Gallop & Weschle, 2019
- One exception being random errors
  - Even if we know the magnitude of the error mechanism, we will not be able to estimate each case's true value

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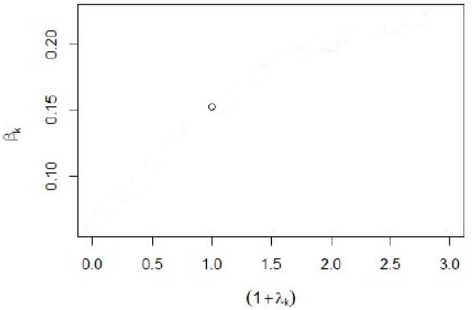
5. $\hat{\beta}_{SIMEX}$ can now be calculated by extrapolating to $\lambda_k = -1$

SIMEX



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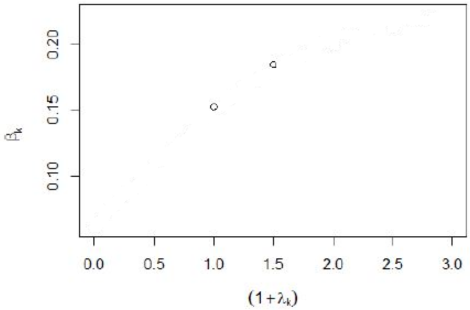

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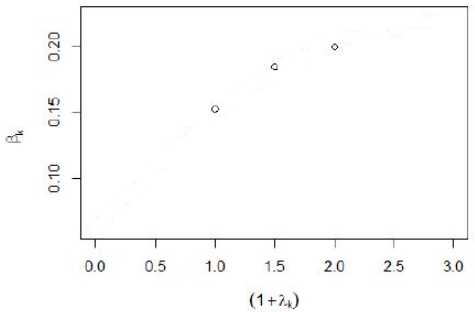

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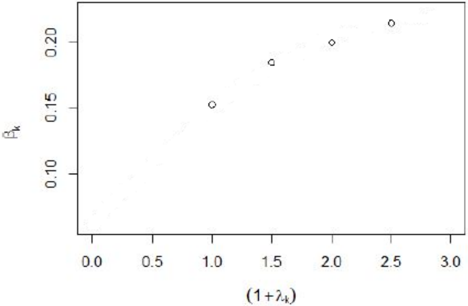

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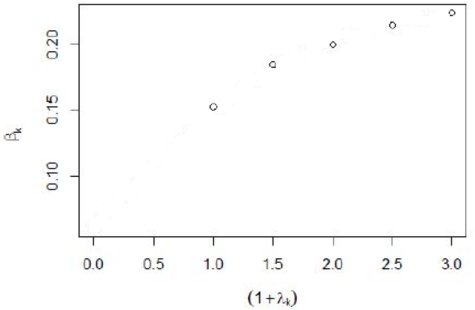

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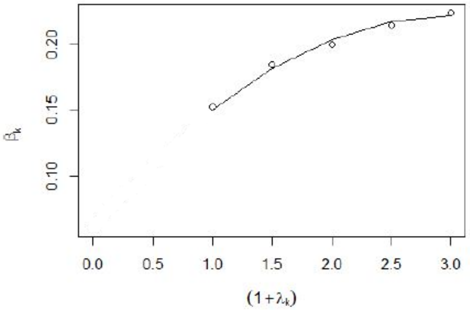

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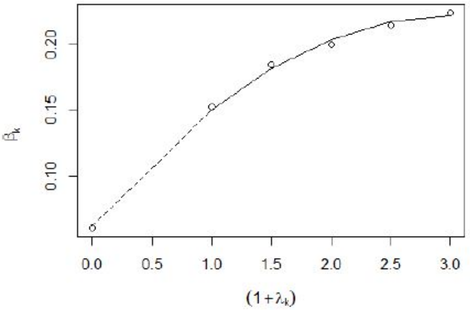

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- The quality of the adjustment depends on:
  - The accuracy with which we define the measurement error mechanism
  - Choosing the right extrapolation function
- A very flexible approach
  - An R package (simex) with built-in commands to explore general cases (e.g. classical errors, misclassification)
  - New packages exploring other measurement error forms (e.g. multiplicative errors)
  - Not perfectly flexible though, we can only explore pre-established measurement error forms
  - Only explores the impact of measurement error when the variable affected is the predictor of interest

Bayesian Adjustments

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  - The former reflects the substantive relationship that we want to estimate
  - The latter can reflect any form of measurement error that we can express algebraically
- These two (or more) models are estimated simultaneously
  - Using Markov chain Monte Carlo (MCMC) methods
  - We obtain a 'posterior distribution' for each estimate included in our models
  - This reflects the probability distribution of an estimate given: i) the models we are using, ii) the data that we observe, and iii) any prior knowledge we might want to include

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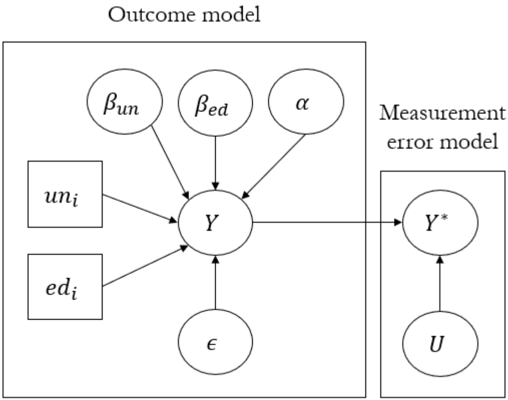
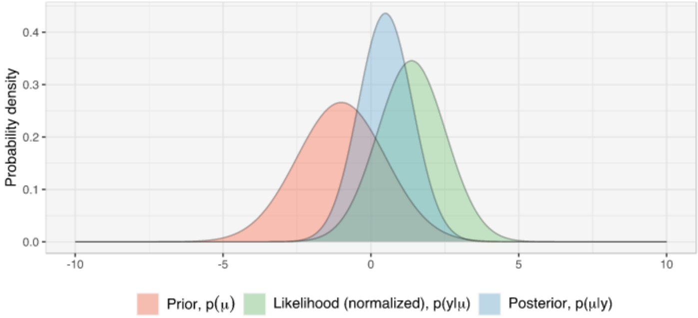


Bayesian Statistics: Key Concepts

- Prior distribution
  - Represents our believe about the parameters to be estimated before observing data
  - We can use diffuse/vague priors to indicate that we are agnostic
- Likelihood function
  - Describes the probability of observing the data given the parameters
  - This is what frequentist statistics is based upon
- Posterior distribution
  - The updated distribution of parameters after considering the data

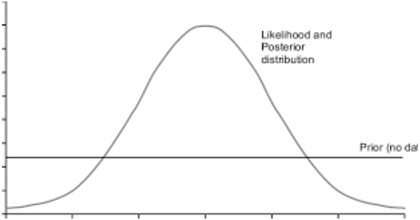
Bayesian Statistics: Key Concepts

The *posterior distribution* as a combination of the *likelihood function* and *prior distribution*



Source: Bolstad (2018)

When using diffuse priors the posterior distribution equals the likelihood function



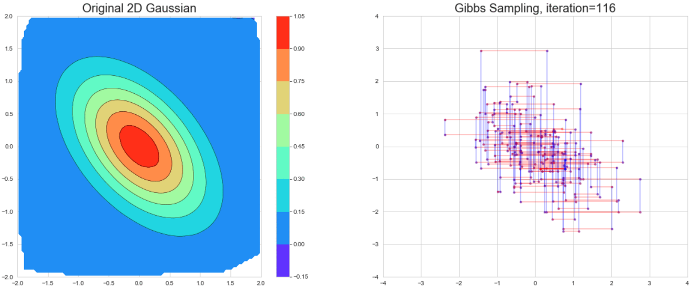
Source: Ghazoul and McAllister (2003)

Key Concepts: MCMC Estimation

- MCMC is a class of algorithms used for sampling from complex probability distributions
- In Bayesian statistics, MCMC is employed to draw samples from the posterior distribution
- They explore the parameter space and approximate the posterior distribution without requiring explicit solutions
  - *Metropolis-Hastings algorithm* and *Gibbs sampling* are common MCMC techniques

MCMC Estimation: Gibbs Sampling

Gibbs sampling algorithm approximating a Gaussian distribution

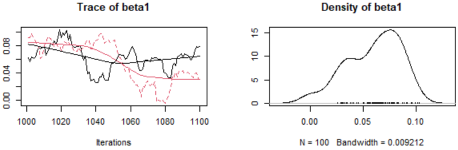


Source: Dey (2020)

Key Concepts: Convergence

- The sampled values will only represent the target distribution accurately after the MCMC chain/s have converged
- We can provide sensible values to initiate the MCMC chain to facilitate convergence
- Still a range of first values should be discarded (*burn-in*) to ensure we only infer from values after the chains converged
- To assess convergence we can use traceplots and the Gelman-Rubin diagnostic

MCMC chains that have not converged



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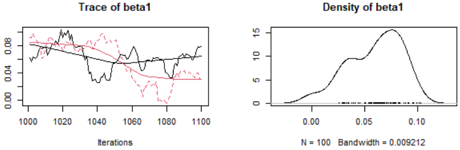
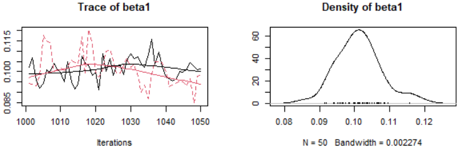


MCMC chains that might have converged but the sample is not big enough



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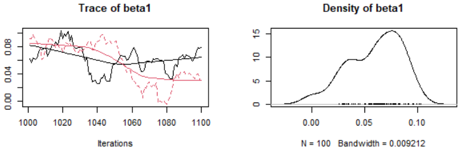

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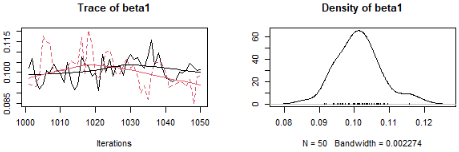
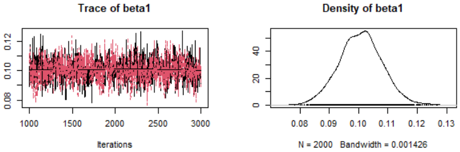


MCMC chains that have converged and can estimate the posterior distribution precisely



Flexibility of Bayesian Adjustments

- Probably the method with the steepest learning curve
  - We need to use Bayesian software (e.g. Stan, JAGS)
  - And keep learning about Bayesian inference
- But can be expanded in lots of different ways
  - Informative priors, we can incorporate any subjective knowledge we possess about any of the parameters to be estimated
  - Can adjust for measurement error and missing data simultaneously
  - And any other questionable assumptions (unobserved confounders, heteroskedsticity, etc.)

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